AQA

A-level MATHEMATICS MM03

Mechanics

Mark scheme

June 2019

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196AMM03/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

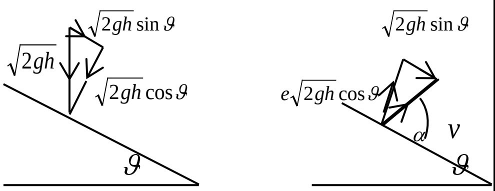
Otherwise we require evidence of a correct method for any marks to be awarded.

Question	Solution	Mark	Total	Comments
1	Dimension of g is $\mathrm{LT^{-2}}$ Dimension of s is L Dimension of h is L Dimension of m_1 and m_2 is M Dimension of $g[s(m_1+m_2)+\frac{hm_1^2}{m_1+m_2}]$ is $\mathrm{LT^{-2}[LM+\frac{LM^2}{M}] \cong ML^2T^{-2}+ML^2T^{-2}}$ $\cong \mathrm{ML^2T^{-2}}$ which is not a force, so not consistent	B1 M1 A1 B1	4	B1: dimensions of the five quantities M1:Correct substitution of dimensions A1: Correct simplification B1: Statement A1 mark: Do **not** condone numerical coefficients, e.g. $\frac{5}{2}\mathrm{ML^2T^{-2}}$, etc.
	Total		4	

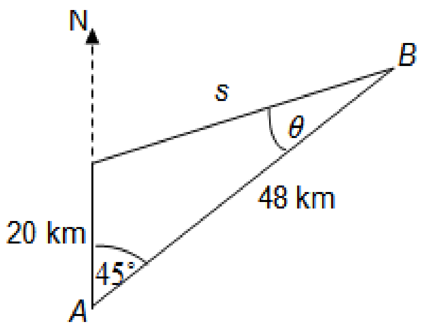
Question	Solution	Mark	Total	Comments
2 (a)	$x = u\cos\alpha t$	M1		
	$y = u\sin\alpha t - \frac{1}{2}gt^2$	M1		
	$t = \frac{x}{u\cos\alpha}$	A1		
	$y = u\sin\alpha \times \frac{x}{u\cos\alpha} - \frac{1}{2}(9.8)\left(\frac{x}{u\cos\alpha}\right)^2$	m1		
	$y = x\tan\alpha - \frac{4.9x^2}{u^2\cos^2\alpha}$	A1	5	AG
(b) (i)	$-s = s\tan 55^\circ - \frac{4.9s^2}{21^2\cos^2 55^\circ}$	M1		
	$s = \frac{(1+\tan 55)21^2\cos^2 55^\circ}{4.9}$	m1		
	$s = 71.9$	A1	3	
(b) (ii)	$\dot{x} = 21\cos 55^\circ$	M1		
	$\dot{y} = 21\sin 55^\circ - 9.8 \times \frac{71.9}{21\cos 55^\circ}$	M1		
	$= -41.3$	A1		
	$\tan^{-1}\frac{-41.3}{21\cos 55^\circ}$	m1		PI by correct answer
	$= -74^\circ$	E1		AWRT
	At an angle of depression of 74°		5	OE statement needed for E1
	Total		**13**	

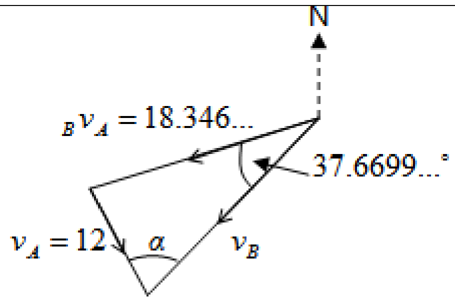
Question	Solution	Mark	Total	Comments
3 (a)	$I = \int_0^3 (3t+1)\,dt$	M1		Condone missing limits
	$= \left[\frac{3}{2}t^2 + t\right]_0^3$	m1		For correct integration only
	$= \frac{33}{2}$ or 16.5 Ns	A1	3	Condone missing units
(b)	$\frac{33}{2} = 0.5v - 0.5(4)$	M1		Impulse/momentum equation
	$v = 37 \text{ ms}^{-1}$	A1F	2	FT on their impulse from part (a)
(c)	$\int_0^T (3t+1)\,dt = 0.5(20) - 0.5(4)$ $\left[\frac{3}{2}t^2 + t\right]_0^T = 0.5(20) - 0.5(4)$	M1		Correct impulse-momentum equation, condone missing limits
	$3T^2 + 2T - 16 = 0$	A1		
	$(3T+8)(T-2) = 0$ or $T = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-16)}}{2(3)}$	m1		
	$T = 2$ s $\left(T = -\frac{8}{3} \text{ s impossible}\right)$	A1	4	(a) Condone missing dt (c) Rejecting impossible time PI
	Total		9	

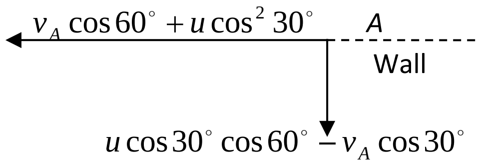
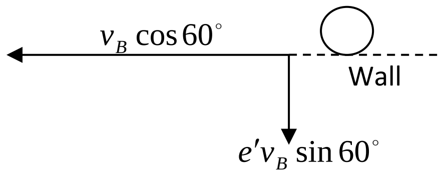
Question	Solution	Mark	Total	Comments
4 (a)	$m(4u)+3m(2u)=mv_A+3mv_B$	M1 A1		M1: four momentum terms A1:all correct OE
	$\dfrac{v_B-v_A}{4u-2u}=e$	M1		
	$v_A+3v_B=10u$ $v_B-v_A=2ue$	A1		For both equations without *m* and without fraction OE
	$4v_B=2ue+10u$			
	$v_B=\dfrac{u}{2}(e+5)$ OE			
	$v_A=\dfrac{u}{2}(e+5)-2ue$	m1		m1: complete solution
	$v_A=\dfrac{u}{2}(-3e+5)$ OE	A1	6	A1: both v_A and v_B
(b)	$e\le 1 \Rightarrow \text{Max } v_B=\dfrac{u}{2}(1+5)$	M1		Sight of $e\le 1$ (OE) needed
	or Max $v_B=3u$	A1	2	FT their v_B CAO
(c)	$(I=)\pm 3m.\dfrac{u}{2}(\dfrac{2}{3}+5)\pm 3m.2u$	M1		M1: a difference of two momentums FT their velocity from part (a)
	$=\dfrac{5mu}{2}$ or $2.5mu$	A1F		A1F for their *'Final B – Initial B'*
	(c) Alternative: $(I=)\pm m\times 4u\pm m\times\dfrac{u}{2}\left(-3\times\dfrac{2}{3}+5\right)$ (M1) for a difference of two momentums FT their velocity from part (a) (A1F) for their 'Initial A - Final A' $=\dfrac{5mu}{2}$ or $2.5mu$ (A1)	A1	3	
	Total		11	

Question	Solution	Mark	Total	Comments
5 (a)	Ball and plane are smooth $\Rightarrow$ mutual reaction acts perpendicular to the plane $\Rightarrow$ no change in momentum/velocity parallel to plane	**E2, 1**	**2**	E1 for each implied statement
(b)	 Before After $\sqrt{2gh}$ $\sqrt{2gh}\sin\vartheta$ $e\sqrt{2gh}\cos\vartheta$	 **B1** **B1** **B1**	**3**	
(c)	(At B,) $\quad 0 = e\sqrt{2gh}\cos\vartheta\, t - \frac{1}{2} g\cos\vartheta\, t^2$ $t = \dfrac{2e\sqrt{2gh}\cos\vartheta}{g\cos\vartheta}$ or $\dfrac{2e\sqrt{2gh}}{g}$ $x = \sqrt{2gh}\sin\vartheta\, t + \frac{1}{2} g\sin\vartheta\, t^2$ $AB = \dfrac{\sqrt{2gh}\sin\vartheta 2e\sqrt{2gh}}{g} + \dfrac{g\sin\vartheta 4e^2 2gh}{2g^2}$ $AB = \dfrac{4ghe\sin\vartheta}{g} + \dfrac{8g^2he^2\sin\vartheta}{2g^2}$ $AB = 4he\sin\vartheta + 4he^2\sin\vartheta$ $AB = 4he(e+1)\sin\vartheta$	**M1 A1** **m1** **M1 A1** **A1F** **A1**	**7**	A0 for sign error A0 for sign error Elimination of *t*, one slip OE OE AG, must be convinced

	5(c) *Alternative* (At B,) $0 = v\sin\alpha\, t - \frac{1}{2}gt^2\cos\vartheta$ M1 $t = \frac{2v\sin\alpha}{g\cos\vartheta}$ m1 $x = v\cos\alpha t + \frac{1}{2}gt^2\sin\vartheta$ M1 $AB = v\cos\alpha\left(\frac{2v\sin\alpha}{g\cos\vartheta}\right) + \frac{1}{2}g\left(\frac{2v\sin\alpha}{g\cos\vartheta}\right)^2\sin\vartheta$ A1 $AB = \frac{2v^2\sin\alpha\cos\alpha}{g\cos\vartheta} + \frac{2v^2\sin^2\alpha\sin\vartheta}{g\cos^2\vartheta}$ $\sin\alpha = \frac{\sqrt{2gh}\,e\cos\vartheta}{v}$, $\cos\alpha = \frac{\sqrt{2gh}\sin\vartheta}{v}$ B1 (for both) $AB = \frac{2v^2 \times \frac{\sqrt{2gh}\,e\cos\vartheta}{v} \times \frac{\sqrt{2gh}\sin\vartheta}{v}}{g\cos\vartheta} + \frac{2v^2\left(\frac{\sqrt{2gh}\,e\cos\vartheta}{v}\right)^2\sin\vartheta}{g\cos^2\vartheta}$ m1 $AB = 4he\sin\vartheta + 4he^2\sin\vartheta$ $AB = 4he(e+1)\sin\vartheta$ A1 AG, must be convinced			
	Total		**12**	

Question	Solution	Mark	Total	Comments
6 (a)		B1		B1: Diagram, PI by correct method
	$s^2 = 20^2 + 48^2 - 2\times 20\times 48\cos 45^\circ$	M1		M1: Cosine rule to find the relative distance
	$s = 36.6927...$ km			
	${}_Bv_A = \frac{36.6927...}{2}$	m1		m1:Dividing their distance by 2
	$= 18.346...$ km h^{-1}	A1		A1:AWRT 18.3 or 18.4
	$\frac{\sin\theta}{20} = \frac{\sin 45}{36.6927...}$	M1		M1: Sine rule to find θ
	$\theta = 22.669....^\circ$			
	Bearing: 248°	A1	6	A1:AWRT 248°

(b) (i)		**B1**		B1: Diagram, PI by correct method
	$\frac{\sin\alpha}{18.346...} = \frac{\sin 37.6699...^\circ}{12}$	**M1**		M1: Sine rule to find α FT from (a)
	$\alpha = 69.1161...^\circ$, $110.8838...^\circ$	**A1**		A1:Correct values for α (PI by correct bearings)
	Bearings: $210^\circ - 69.1161...^\circ$ or $210^\circ - 110.8838...^\circ$ $= 141^\circ$ or 099°	**A1**	**4**	A1:Correct bearings , CAO
(ii)	$\frac{v_B}{\sin(180^\circ - 37.6699...^\circ - 69.1161...^\circ)} = \frac{12}{\sin 37.6699...^\circ}$ OE	**M1**		M1: Sine rule to find v_B , (FT angle from (b)(i))
	$v_B = 18.8 \text{ km h}^{-1}$	**A1**	**2**	A1: AWRT 18.8 km h^{-1} , CAO
	Total		**12**	

Question	Solution	Mark	Total	Comments
7 (a)	Along the line of centres:			
	CLM: $u\cos 60^\circ = v_A + v_B$ OE	M1 A1		Allow sign error for M1, A1 for all correct
	Restitution: $eu\cos 60^\circ = v_B - v_A$	M1 A1		
	$2v_B = (1+e)u\cos 60^\circ$ OE			
	$v_B = \frac{1}{4}u(1+e)$	A1		AG above line oe needed
	$v_A = \frac{1}{4}u(1-e)$ OE	A1		
	Perpendicular to line of centres: $v_A' = u\cos 30^\circ$ OE	A1	7	
(b)		M1 A1		OE
		M1 A1		OE
	$\frac{u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ}{v_A\cos 60^\circ + u\cos^2 30^\circ} = \frac{e'v_B\sin 60^\circ}{v_B\cos 60^\circ}$	m1		OE dependent on both M1s
	$\frac{u\times\frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{4}u(1-e)\times\frac{\sqrt{3}}{2}}{\frac{1}{4}u(1-e)\times\frac{1}{2} + u\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{e'\times\frac{\sqrt{3}}{2}}{\frac{1}{2}}$	A1	7	OE
	$e' = \frac{2u - u + eu}{u - eu + 6u}$			
	$e' = \frac{1+e}{7-e}$	A1		AG
	Total		**14**	