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A-level MATHEMATICS MM03

Mechanics

Mark scheme

June 2019

Version: v1.0 Final



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

Μ	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
А	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and
	accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
С	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Question	MARK SCHEME – A-LE	/EL MATH	EMATICS	- MM03 - JUNE 20
1	Dimension of g is LT ⁻² Dimension of s is L Dimension of h is L Dimension of m_1 and m_2 is M Dimension of $g[s(m_1 + m_2) + \frac{hm_1^2}{m_1 + m_2}]$ is LT ⁻² [LM + $\frac{LM^2}{M}$] \cong ML ² T ⁻² + ML ² T ⁻² \cong ML ² T ⁻² which is not a force, so not consistent	M1 A1 B1		B1: dimensions of the five quantities M1:Correct substitution of dimensions A1: Correct simplification B1: Statement
			4	A1 mark: Do not condone numerical coefficients, e.g. $\frac{5}{2}$ ML ² T ⁻² , etc.
	Total		4	

	MARK SCHEME – A-LEVE	EL MATHE	MATICS -	MM03 - JUNE 20-
Question	Solution	Mark	Total	Commonts
			Total	Comments
2 (a)	$x = u \cos \alpha i$	IVIT		
	$y = u\sin\alpha t - \frac{1}{2}gt^2$	M1		
	$t = \frac{x}{\mu \cos \alpha}$	A1		
	$y = u \sin \alpha \times \frac{x}{u \cos \alpha} - \frac{1}{2} (9.8) \left(\frac{x}{u \cos \alpha}\right)^2$	m1		
	$y = x \tan \alpha - \frac{4.9x^2}{u^2 \cos^2 \alpha}$	A1	5	AG
(b) (i)				
	$-s = s \tan 55^{\circ} - \frac{4.9s^2}{21^2 \cos^2 55^{\circ}}$	M1		
	$s = \frac{(1 + \tan 55)21^2 \cos^2 55^\circ}{1 + \tan 55}$	m1		
	4.9			
	s = /1.9	A1		
			3	
(b) (ii)				
	$\dot{x} = 21\cos 55^{\circ}$	M1		
	$\dot{y} = 21\sin 55^\circ - 9.8 \times \frac{71.9}{21\cos 55^\circ}$	M1		
	= -41.3	Δ1		
	\tan^{-1} <u>-41.3</u>	m1		PI by correct
	$21\cos 55^{\circ}$ $- 74^{\circ}$	E4		answer
	/ 4	EI		
	At an angle of depression of 74°		5	OE statement needed for E1
	Total		13	
			1	

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	MARK SCHEME – A-LEV	EL MATH	EMATICS	- MMU3 - JUNE 201 - Marting
Question	Solution	Mark	Total	Comments
3 (a)	$\mathbf{I} = \int_{0}^{3} (3t+1) \mathrm{d}t$	M1		Condone missing limits
	$= \left[\frac{3}{2}t^2 + t\right]_0^3$	m1		For correct integration
	$=\frac{33}{2}$ or 16.5 Ns	A1	3	Condone missing units
(b)	$\frac{33}{2} = 0.5v - 0.5(4)$	M1		Impulse/momentum equation
	$v = 37 \text{ ms}^{-1}$	A1F	2	FT on their impulse from part (a)
(c)	$\int_{0}^{T} (3t+1) dt = 0.5(20) - 0.5(4)$	M1		Correct impulse-
	$\left[\frac{3}{2}t^2 + t\right]_0^T = 0.5(20) - 0.5(4)$			momentum equation, condone missing limits
	$3T^2 + 2T - 16 = 0$	A1		
	$(3T+8)(T-2) = 0$ or $T = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-16)}}{2(3)}$	m1		
	T = 2 s	A1		
	$\left(T = -\frac{3}{3} \text{ s impossible}\right)$		4	
				(a) Condone missing dt
				(c) Rejecting impossible time PI
	Total		9	

	MARK SCHEME – A-I EVEL MA	THFMAT	ICS – MN	103 - JUNE 20 Mm
				Aths.
Question	Solution	Mark	Total	Comments
4 (a)	$m(4u) + 3m(2u) = mv_A + 3mv_B$ $\frac{v_B - v_A}{4u - 2u} = e$	M1 A1 M1		M1: four momentum terms A1:all correct OE
	$v_A + 3v_B = 10u$ $v_B - v_A = 2ue$ $4v_B = 2ue + 10u$	} A1		For both equations without <i>m</i> and without fraction OE
	$v_B = \frac{u}{2}(e+5) \qquad \text{OE}$ $v_A = \frac{u}{2}(e+5) - 2ue$			
	$v_A = \frac{2}{2}(-3e+5)$ 2 <i>u</i> c $v_A = \frac{u}{2}(-3e+5)$ OE	m1 A1	6	m1: complete solution A1: both v_A and v_B
(b)	$e \le 1 \implies \text{Max } v_B = \frac{u}{2}(1+5)$ or Max $v_B = 3u$	M1	0	Sight of $e \le 1$ (OE) needed FT their v_B
(c)	$(I =) \pm 3m \cdot \frac{u}{2}(\frac{2}{3} + 5) \pm 3m \cdot 2u$	A1	2	CAO
	$=\frac{5mu}{2}$ or 2.5mu	M1		M1: a difference of two momentums FT their velocity from part (a)
	(c) Alternative:	A1F		A1F for their ' <i>Final B –</i> Initial B'
	$(I =) \pm m \times 4u \pm m \times \frac{u}{2} \left(-3 \times \frac{2}{3} + 5 \right)$ (M1) for a difference of two momentums FT their velocity	A1	3	
	$(A1F) \text{for their 'Initial A - Final A'} = \frac{5mu}{2} \text{or } 2.5mu \tag{A1}$			
	T =4=1		44	
	lotal		11	

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	MARK SCHEME - A-LEV		EMATICS	- MM02 - JONE 207	naths clou
Question	Solution	Mark	Total	Comments	4 <u>4</u> .0
5 (a)	Ball and plane are smooth ⇒ mutual reaction acts perpendicular to the plane ⇒ no change in momentum/velocity parallel to plane	E2, 1	2	E1 for each implied statement	
(b)	$ \frac{\sqrt{2gh}}{\sqrt{2gh}} \sin \theta \qquad \sqrt{2gh} \sin \theta \\ \frac{\sqrt{2gh}}{\sqrt{2gh}} \cos \theta \qquad e\sqrt{2gh} \cos \theta \\ \frac{\sqrt{2gh}}{\sqrt{2gh}} \cos \theta \qquad \frac{\sqrt{2gh}}{\sqrt{2gh}} \cos \theta \\ Before \qquad After $				
	$\sqrt{2gh}$	B1			
	$\sqrt{2gh}\sin \vartheta$	B1			
	$e\sqrt{2gh}\cos\vartheta$	B1	3		
(c)	(At B,) $0 = e\sqrt{2gh}\cos\vartheta t - \frac{1}{2}g\cos\vartheta t^2$	M1 A1		A0 for sign error	
	$t = \frac{2e\sqrt{2gh}\cos\vartheta}{g\cos\vartheta} \text{or} \frac{2e\sqrt{2gh}}{g}$	m1			
	$x = \sqrt{2gh}\sin\vartheta t + \frac{1}{2}g\sin\vartheta t^2$	M1 A1		A0 for sign error	
	$AB = \frac{\sqrt{2gh}\sin \theta 2e\sqrt{2gh}}{g} + \frac{g\sin \theta 4e^2 2gh}{2g^2}$	A1F		Elimination of <i>t</i> , one slip OE	
	$AB = \frac{4ghe\sin\theta}{g} + \frac{8g^2he^2\sin\theta}{2g^2}$ $AB = 4he\sin\theta + 4he^2\sin\theta$			OE	
	$AB = 4he(e+1)\sin\vartheta$	A1	7	AG, must be convinced	

MARK SCHEME – A-LEV	/EL MATH	EMATICS	– MM03 – JU	WWW. My My My	
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5(c) Alternative					COM
(At B,) $0 = v \sin \alpha t - \frac{1}{2} g t^2 \cos \theta$	M1				
$t = \frac{2v\sin\alpha}{g\cos\theta}$	m1				
$x = v \cos \alpha t + \frac{1}{2}gt^2 \sin \vartheta$	M1				
$AB = v\cos\alpha \left(\frac{2v\sin\alpha}{g\cos\vartheta}\right) + \frac{1}{2}g\left(\frac{2v\sin\alpha}{g\cos\vartheta}\right)^2\sin\vartheta$	A1				
$AB = \frac{2v^2 \sin \alpha \cos \alpha}{g \cos \theta} + \frac{2v^2 \sin^2 \alpha \sin^2 \theta}{g \cos^2 \theta}$					
$\sin \alpha = \frac{\sqrt{2gh} e \cos \theta}{v}$ $\cos \alpha = \frac{\sqrt{2gh} \sin \theta}{v}$	B1 (for b	both)			
$AB = \frac{2v^2 \times \frac{\sqrt{2gh} e \cos \theta}{v} \times \frac{\sqrt{2gh} \sin \theta}{v}}{g \cos \theta} + \frac{2v^2 \left(\frac{\sqrt{2gh} e \cos \theta}{v}\right)^2 \sin \theta}{g \cos^2 \theta}$	ml				
$AB = 4he\sin\vartheta + 4he^2\sin\vartheta$ $AB = 4he(e+1)\sin\vartheta$	A1 AG,	must be convince	d		
Total	,	12			

	MARK SCHEME – A-LEV	/EL MATH	EMATICS	- MM03 - JUNE 20_ MR	My Math
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Question	Solution	Mark	Total	Comments	
6 (a)					
	N B 8 θ 48 km 20 km				
	A 45-2	B1		B1: Diagram, PI by correct method	
	$s^2 = 20^2 + 48^2 - 2 \times 20 \times 48 \cos 45^\circ$	M1		M1: Cosine rule to find the relative distance	
	<i>s</i> = 36.6927 km				
	$_{B}v_{A} = \frac{36.6927}{2}$	m1		m1:Dividing their distance by 2	
	$= 18.346 \text{ km h}^{-1}$	A1		A1:AWRT 18.3 or 18.4	
	$\frac{\sin\theta}{20} = \frac{\sin 45}{36.6927}$ $\theta = 22.669^{\circ}$	М1		M1: Sine rule to find θ	
	Bearing: 248°	A1	6	A1:AWRT 248°	

	MARK SCHEME – A-LEV	/EL MATH	EMATICS	- MM03 - JUNE 20- MR03	MA NOTING
(b) (i)	$BV_A = 18.346$ 37.6699°	B1		B1: Diagram, PI by	SCIOUD.COM
	$\frac{\sin \alpha}{18.346} = \frac{\sin 37.6699^{\circ}}{12}$	М1		correct method M1: Sine rule to find α	
	$\alpha = 69.1161^{\circ}$, 110.8838°	A1		FT from (a) A1:Correct values for α (Pl by correct bearings)	
	Bearings: $210^{\circ} - 69.1161^{\circ}$ or $210^{\circ} - 110.8838^{\circ}$ = 141° or 099°	A1	4	A1:Correct bearings , CAO	
(ii)	$\frac{v_B}{\sin(180^\circ - 37.6699^\circ - 69.1161^\circ)} = \frac{12}{\sin 37.6699^\circ} \text{OE}$	М1		M1: Sine rule to find v_B , (FT angle from (b)(i))	
	$v_B = 18.8$ km h ⁻¹	A1	2	A1: AWRT 18.8 km h ⁻¹ , CAO	
	Total		12		

$\frac{\text{Direction}}{\text{7 (a)}} \frac{\text{Solution}}{\text{Along the line of centres:}} \\ \frac{\text{CLM: } u \cos 60^{\circ} = v_{n} + v_{n}}{\text{ClM: } u \cos 60^{\circ} = v_{n} + v_{n}} OE \\ \frac{\text{CLM: } u \cos 60^{\circ} = v_{n} + v_{n}}{2v_{n} = (1 + e)v \cos 60^{\circ}} OE \\ \frac{v_{n} = \frac{1}{4}u(1 + e)}{v_{n} = \frac{1}{4}u(1 - e)} OE \\ \frac{v_{n} = \frac{1}{4}u(1 - e)}{\frac{1}{4}u(1 - e)} OE \\ \frac{\text{A1}}{\text{A1}} \frac{\text{A1}}{\text{A1}} \frac{\text{A2}}{\text{A1}} \frac{\text{A3}}{\text{A1}} \\ \frac{\text{A2}}{\text{A1}} \frac{\text{A3}}{\text{A1}} \frac{\text{A3}}{\text{A1}} \frac{\text{A3}}{\text{A1}} \\ \frac{\text{A4}}{1} \frac{\text{A6}}{1} \frac{\text{A6}}{1} \frac{\text{A6}}{1} \\ \frac{v_{n} \cos 60^{\circ} + u \cos^{2} 30^{\circ}}{v_{n} \cos 60^{\circ} - v_{n} \cos 30^{\circ}} OE \\ \frac{u \cos 30^{\circ} \cos 60^{\circ} + u \cos^{2} 30^{\circ}}{v_{n} \cos 60^{\circ}} \frac{\text{A1}}{\frac{v_{n} \sin 60^{\circ}}{v_{n} \cos 60^{\circ}}} \frac{\text{A1}}{\frac{1}{4}u(1 - e)} \frac{\text{A1}}{2} \frac{1}{2} $				MATICC	MMM2 HINE 20 M
AlestionNumber of contrestsContrents7 (a)Along the line of centres: CLM: $ucos60^{\circ} = v_{A} + v_{x}$ OEM1 A1Allow sign error for M1, A1 for all correctRestitution: $ucos60^{\circ} = v_{x} - v_{A}$ OEM1 A1AG above line oe needed $2v_{x} = (1 + e)vcos60^{\circ}$ OEA1AG above line oe needed $v_{x} = \frac{1}{4}u(1 + e)$ $v_{x} = \frac{1}{4}u(1 - e)$ OEA1AG above line oe needed(b) $\underbrace{v_{x} \cos 60^{\circ} + u \cos^{2} 30^{\circ}}_{u \cos 30^{\circ} \cos 60^{\circ} - v_{A} \cos 30^{\circ}}_{Wall}$ M1 A1OE(b) $\underbrace{v_{x} \cos 60^{\circ} + u \cos^{2} 30^{\circ}}_{v_{x} \cos 60^{\circ} - v_{A} \cos 30^{\circ}}_{Wall}$ M1 A1OE(c) $\underbrace{v_{x} \cos 60^{\circ} + u \cos^{2} 30^{\circ}}_{v_{x} \cos 60^{\circ}}_{Wall}$ M1 A1OE $u \cos 30^{\circ} \cos 60^{\circ} - v_{A} \cos 30^{\circ}_{v_{x}} = \frac{e^{i}v_{x} \sin 60^{\circ}}{v_{a} \cos 60^{\circ}}_{v_{x} $	Question		MATHE		Commonts
7 (a) Along the line of centres: CLM: $u\cos 60^{\circ} = v_{A} + v_{B}$ OE M1 A1 Allow sign error for M1, A1 for all correct Restitution: $eu\cos 60^{\circ} = v_{A} - v_{A}$ M1 A1 Allow sign error for M1, A1 for all correct $v_{a} = \frac{1}{4}u(1+e)$ OE A1		3010101	IVIAI K	TOtal	Comments
CLM: $u\cos 60^\circ = v_A + v_B$ OE M1 A1 Restitution: $eu\cos 60^\circ = v_B - v_A$ M1 A1 $2v_B = (1 + e)u\cos 60^\circ$ OE A1 $v_B = \frac{1}{4}u(1 - e)$ OE A1 Perpendicular to line of centres: $v_A' = u\cos 30^\circ$ OE A1 $u\cos 30^\circ\cos 60^\circ + u\cos^2 30^\circ$ A M1 A1 $u\cos 30^\circ\cos 60^\circ + u\cos^2 30^\circ$ A M1 $u\cos 30^\circ\cos 60^\circ + u\cos^2 30^\circ$ A A A $u\cos 30^\circ\cos 60^\circ + u\cos^2 30^\circ$ A A $u\cos 30^\circ\cos 60^\circ + u\cos^2 30^\circ$ A $u\cos 30^\circ \cos 60^\circ + u\cos^2 30^\circ$ A $u\cos 30^\circ \cos 60^\circ + u\cos^2 30^\circ$ A A1 $du = \frac{u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ}$ A1 $du = \frac{u\cos 30^\circ - v_A\cos 30^\circ}{u_B\cos 60^\circ$	7 (a)	Along the line of centres:			
Restitution: $u \cos 60^\circ = v_u - v_x$ M1 A1 AG above line oe needed $v_u = \frac{1}{4}u(1 + e)$ OE A1 AG above line oe needed $v_u = \frac{1}{4}u(1 - e)$ OE A1 AG above line oe needed Perpendicular to line of centres: $v'_x = u \cos 30^\circ$ OE A1 7 (b) $\underbrace{v_x \cos 60^\circ + u \cos^2 30^\circ}_{u \cos 30^\circ} = \underbrace{w_x \cos 30^\circ}_{w_x \cos 30^\circ}$ M1 A1 OE $u \cos 30^\circ \cos 60^\circ + u \cos^2 30^\circ$ $\underbrace{w_{all}}_{e'v_x} \sin 60^\circ$ M1 A1 OE $\frac{u \cos 30^\circ \cos 60^\circ - v_x \cos 30^\circ}{v_x \cos 60^\circ + u \cos^2 30^\circ} = \underbrace{w_x \sin 60^\circ}_{v_x \cos 60^\circ}$ M1 A1 OE $\frac{u \cos 30^\circ \cos 60^\circ - u \cos^2 30^\circ}{v_x \cos 60^\circ + u \cos^2 30^\circ} = \underbrace{w_x \frac{\sqrt{3}}{2}}_{\frac{1}{2}} = \frac{1}{\frac{1}{2}}$ A1 OE OE $\frac{u \cos 30^\circ \cos 60^\circ - u \cos^2 30^\circ}{v_x \cos 60^\circ + u \cos^2 30^\circ} = \underbrace{w_x \frac{\sqrt{3}}{2}}_{\frac{1}{2}} = \underbrace{w \sqrt{3}}_{\frac{1}{2}}$ A1 OE OE $\frac{u \cos 30^\circ \cos 60^\circ - u \cos^2 30^\circ}{v_x \cos 60^\circ + u \cos^2 30^\circ} = \underbrace{w \sqrt{3}}_{\frac{1}{2}}$ A1 A1 OE $\frac{u + w \sqrt{3}}{v_x \cos 60^\circ + u \cos^2 30^\circ} = \underbrace{w \sqrt{3}}_{\frac{1}{2}}$ A1 A1 A2 $\frac{v + u + eu}{u + eu + 6u}$ $\frac{v + 1}{2}$ A1 A2 A3 $\frac{v + 1}{4} = \frac{1 + eu}{1 - eu}$ A1 A3		$CLM: u\cos 60^\circ = v_A + v_B \qquad \qquad OE$	M1 A1		Allow sign error for M1, A1 for all correct
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		Restitution : $eu \cos 60^\circ = v_B - v_A$	M1 A1		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$2v_B = (1+e)u\cos 60^\circ \qquad \qquad OE$			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$v_{B} = \frac{1}{4}u(1+e)$	A1		AG above line oe
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$v_A = \frac{1}{4}u(1-e) \qquad \qquad OE$	A1		needed
(b) $v_A \cos 60^\circ + u \cos^2 30^\circ$ A Wall $u \cos 30^\circ \cos 60^\circ + v_A \cos 30^\circ$ M1 A1 OE $v_B \cos 60^\circ + v_A \cos 30^\circ$ M1 OE $v_B \cos 60^\circ + v_A \cos 30^\circ$ M1 OE $\frac{u \cos 30^\circ \cos 60^\circ - v_A \cos 30^\circ}{v_A \cos 60^\circ + u \cos^2 30^\circ} = \frac{e'v_B \sin 60^\circ}{v_B \cos 60^\circ}$ M1 OE $\frac{u \times \sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{4} u(1 - e) \times \frac{\sqrt{3}}{2} = \frac{e' \times \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ A1 OE $e' = \frac{2u - u + eu}{u - eu + 6u}$ A1 OE $e' = \frac{1 + e}{7 - e}$ A1 AG		Perpendicular to line of centres : $v'_A = u \cos 30^\circ$ OE	A1	7	
(b) $v_A \cos 60^\circ + u \cos^2 30^\circ A$ Wall $u \cos 30^\circ \cos 60^\circ - v_A \cos 30^\circ$ $v_B \cos 60^\circ - v_A \cos 30^\circ$ $\frac{u \cos 30^\circ \cos 60^\circ - v_A \cos 30^\circ}{v_A \cos 60^\circ + u \cos^2 30^\circ} = \frac{e'v_B \sin 60^\circ}{v_B \cos 60^\circ}$ $\frac{u \times \sqrt{3}}{v_A \cos 60^\circ + u \cos^2 30^\circ} = \frac{e'v_B \sin 60^\circ}{v_B \cos 60^\circ}$ $\frac{u \times \sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{4} u (1 - e) \times \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{e' \times \sqrt{3}}{\frac{2}{2}}$ $\frac{1}{4} u (1 - e) \times \frac{1}{2} + u \left(\frac{\sqrt{3}}{2} \right)^2 = \frac{e' \times \sqrt{3}}{\frac{1}{2}}$ $e' = \frac{2u - u + eu}{u - eu + 6u}$ $e' = \frac{1 + e}{7 - e}$ A1 OE OE OE OE M1 A1 OE OE A1 A2 A3 A4 A6				/	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	(b)	$\checkmark \frac{v_A \cos 60^\circ + u \cos^2 30^\circ}{\text{Wall}}$	M1 A1		OE
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$u\cos 30^\circ \cos 60^\circ - v_A\cos 30^\circ$			
$\frac{u\cos 30^{\circ}\cos 60^{\circ} - v_{A}\cos 30^{\circ}}{v_{A}\cos 60^{\circ} + u\cos^{2} 30^{\circ}} = \frac{e'v_{B}\sin 60^{\circ}}{v_{B}\cos 60^{\circ}}$ $\frac{u \times \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{4}u(1 - e) \times \frac{\sqrt{3}}{2}}{\frac{1}{4}u(1 - e) \times \frac{\sqrt{3}}{2}} = \frac{e' \times \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\frac{1}{4}u(1 - e) \times \frac{1}{2} + u\left(\frac{\sqrt{3}}{2}\right)^{2} = \frac{e' \times \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $e' = \frac{2u - u + eu}{u - eu + 6u}$ $e' = \frac{1 + e}{7 - e}$ A1 OE A1 AG		$v_B \cos 60^\circ$ Wall	M1 A1		OE
$\begin{vmatrix} u \cos 30^{\circ} \cos 60^{\circ} - v_{A} \cos 30^{\circ} \\ v_{A} \cos 60^{\circ} + u \cos^{2} 30^{\circ} \\ u \times \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{4}u(1-e) \times \frac{\sqrt{3}}{2} \\ \frac{1}{4}u(1-e) \times \frac{1}{2} + u\left(\frac{\sqrt{3}}{2}\right)^{2} \\ = \frac{e' \times \frac{\sqrt{3}}{2}}{\frac{1}{2}} \\ e' = \frac{2u - u + eu}{u - eu + 6u} \\ e' = \frac{1+e}{7-e} \\ \end{vmatrix}$ $A1$ OE OE $A1$ AG		$e'v_B \sin 60^\circ$			
$\frac{u \times \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{4}u(1-e) \times \frac{\sqrt{3}}{2}}{\frac{1}{4}u(1-e) \times \frac{1}{2} + u\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{e' \times \frac{\sqrt{3}}{2}}{\frac{1}{2}} \qquad A1 \qquad OE$ $e' = \frac{2u - u + eu}{u - eu + 6u}$ $e' = \frac{1+e}{7-e} \qquad A1 \qquad AG$		$\frac{u\cos 30^{\circ}\cos 60^{\circ} - v_{A}\cos 30^{\circ}}{v_{A}\cos 60^{\circ} + u\cos^{2} 30^{\circ}} = \frac{e'v_{B}\sin 60^{\circ}}{v_{B}\cos 60^{\circ}}$	m1		OE dependent on both M1s
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$\frac{u \times \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{4}u(1-e) \times \frac{\sqrt{3}}{2}}{=} = \frac{e' \times \frac{\sqrt{3}}{2}}{2}$	A1		OE
$e' = \frac{2u - u + eu}{u - eu + 6u}$ $e' = \frac{1 + e}{7 - e}$ A1 AG Total I4		$\frac{1}{4}u(1-e) \times \frac{1}{2} + u\left(\frac{\sqrt{3}}{2}\right)^2 \qquad \frac{1}{2}$		7	
$\begin{array}{c c} e = \hline \hline 7 - e \end{array} \qquad \qquad$		$e' = \frac{2u - u + eu}{u - eu + 6u}$			
Total 14		$e^{-}\frac{1}{7-e^{-}}$	A1		AG
		Total		14	